VI Semester B.A./B.Sc. Examination, May 2017
(Semester Scheme)
(Fresh) (CBCS) (2016-17 and Onwards)
MATHEMATICS – VII

Time : 3 Hours Max. Marks : 70

Instruction : *Answer **all** Parts.*

PART – A

1. Answer **any five** questions : **(5×2=10)**

   a) Define a vector space over a field.

   b) For what value of K the vectors (1, 2, 3), (4, 5, 6) and (7, 8, K) are linearly dependent.

   c) Find the matrix of the linear transformation $T : R^2 \rightarrow R^3$ defined by $T(x, y) = (3x - y, 2x + 4y, 5x - 6y)$ w.r.t. the standard basis.

   d) Find the null space of the linear transformation $T : V_3(R) \rightarrow V_2(R)$ defined by $T(x, y, z) = (y - x, y - z)$.

   e) Write scalar factors in cylindrical co-ordinate system.

   f) Solve : $\frac{dx}{yz} = \frac{dy}{zx} = \frac{dz}{xy}$.

   g) Form a partial differential equation by eliminating the arbitrary constants from $z = (x + a)(y + b)$.

   h) Solve : $pq + p + q = 0$.

PART – B

Answer **two full** questions : **(2×10=20)**

2. a) A subset W of a vector space V(F) is a subspace of V(F) if and only if $C_1\alpha + C_2\beta \in W$, for $\alpha, \beta \in W$.

   b) Find the basis and dimension of the subspace spanned by (1, – 1, 0), (0, 3, 1), (1, 2, 1) and (2, 4, 2) in V_3 (R).

OR

3. a) A set of non-zero vectors $(\alpha_1, \alpha_2, ...\alpha_n)$ of vector space V(F) is linearly dependent if and only if one of vectors Say α_K $(2 \le K \le n)$ is expressed as a linear combination of its preceding ones.

   b) Prove that $W = \{(x, y, z) \mid x = y = z\}$ is a subspace of R^3.

4. a) If $T : U \to V$ is a linear transformation then prove that

   i) $T(0) = 0'$ where 0 and $0'$ are the zero vectors of U and V respectively.

   ii) $T(-\alpha) = -T(\alpha)$, $\forall \alpha \in U$.

   b) Find the linear transformation $T : R^2 \to R^2$ such that T (1, 0) = (1, 1) and T (0, 1) = (– 1, 2).

OR

5. a) State and prove rank nullity theorem.

   b) Show that the linear transformation $T : R^3 \to R^3$ given by $T(e_1) = e_1 + e_2$, $T(e_2) = e_1 - e_2 + e_3$ and $T(e_3) = 3e_1 + 4e_3$ is non singular where $\{e_1, e_2, e_3\}$ is the standard basis of R^3.

PART – C

Answer **two full** questions : **(2×10=20)**

6. a) Verify the condition of integrability and solve : $2yzdx + zxdy - xy(1 + z)dz = 0$.

   b) Solve : $(y - z)p + (z - x)q = x - y$.

OR

7. a) Show that cylindrical coordinate system is orthogonal co-ordinate system.

b) Express the vector $\vec{f} = 3y\hat{i} + x^2\hat{j} - z^2\hat{k}$ in cylindrical co-ordinate and find f_ρ, f_ϕ, f_z.

8. a) Solve : $\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$.

b) Solve : $\frac{dx}{y+z} = \frac{dy}{z+x} = \frac{dz}{x+y}$.

OR

9. a) Express the vector $\vec{f} = 3y\hat{i} + 2z\hat{j} + x\hat{k}$ in cylindrical co-ordinates and find f_ρ, f_ϕ, f_z.

b) Express the vector $\vec{f} = z\hat{i} - 2x\hat{j} + y\hat{k}$ in terms of spherical polar co-ordinates and find f_r, f_θ, f_ϕ.

PART – D

Answer **two full** questions : **(2×10=20)**

10. a) Form partial differential equation by eliminating arbitrary function $f(xy + z^2, x + y + z) = 0$.

b) Solve : $p^2 - q^2 = x - y$.

OR

VI Semester B.A./B.Sc. Examination, May/June 2018
(Fresh+Repeaters) (Semester Scheme)
(CBCS) (2016-17 and Onwards)
MATHEMATICS – VII

Time : 3 Hours Max. Marks : 70

Instruction : *Answer **all** Parts.*

PART – A

1. Answer **any five** questions. **(5×2=10)**
   a) In a vector space V over F show that $c \cdot \alpha = 0 \Rightarrow c = 0$ or $\alpha = 0$.
   b) Show that $W = \{(0, 0, z)/z \in R\}$ is a subspace of $V_3(R)$.
   c) Show that the vectors $\alpha_1 = (1, 1, 0)$, $\alpha_2 = (1, 1, 0)$, $\alpha_3 = (1, 0, 0)$ are linearly independent.
   d) Show that $T : V_2(R) \to V_2(R)$ defined by $T(x, y) = (x + y, x - y)$ is a linear transformation.
   e) Write the relation between the Cartesian coordinates and cylindrical coordinates of a point.
   f) Solve $\frac{dx}{zx} = \frac{dy}{yz} = \frac{dz}{xy}$.
   g) Form a partial differential equation by eliminating arbitrary constants from $x^2 + y^2 = (z - c)^2 \tan^2\alpha$, where c and α are arbitrary constants.
   h) Solve $\sqrt{p} + \sqrt{q} = 1$.

PART – B

Answer **two full** questions. **(2×10=20)**

2. a) Show that $V = \left\{ \begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix} \middle| \, x, y \in R \right\}$ is a vector space over R.
   b) State and prove the necessary and sufficient condition for a nonempty subset W of a vector space V(F) to be a subspace of V.

OR

3. a) If V is n-dimensional vector space, show that

   i) any n+1 vectors are linearly dependent.

   ii) no set of n – 1 vectors can span V.

   b) Find the basis and dimension of the subspace spanned by (1, –2, 3), (1, –3, 4), (–1, 1, –2) of the vector space $V_3(R)$.

4. a) Find the linear transformation $T : R^2 \to R^3$ such that T(1, 1) = (0,1, 2), T(–1, 1) = (2, 1, 0).

   b) Given the matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ -1 & 3 \end{bmatrix}$, find the linear transformation $T : V_2(R) \to V_3(R)$ relative to the bases $B_1 = \{(1, 1), (-1, 1)\}$, $B_2 = \{(1, 1, 1), (1, -1, 1), (0, 0, 1)\}$.

OR

5. a) Let $T : V_3(R) \to V_3(R)$ be a linear transformation such that T(1, 0, 0) = (1, 0, 2), T (0, 1, 0) = (1, 1, 0), T(0, 0, 1) = (1, –1, 0). Find the range, null space, rank nullity and hence verify rank-nullity theorem.

   b) Let $T : V \to W$ be a linear transformation. Then show that

   i) R(T) is a subspace of W

   ii) N(T) is a subspace of V

   iii) T is one-one if and only if N(T) = {0}.

PART – C

Answer **two full** questions : **(2×10=20)**

6. a) Verify the condition for integrability and solve $z^2dx + (z^2 - 2yz)\,dy + (2y^2 - yz - xz)\,dz = 0$.

   b) Solve $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$.

OR

7. a) Show that spherical coordinate system is orthogonal curvilinear coordinate system.

   b) Express $\vec{f} = 3xi - 2yzj + x^2zk$ in cylindrical coordinates and find f_ρ, f_ϕ, f_z.

8. a) Solve $\frac{dx}{1+y} = \frac{dy}{1+x} = \frac{dz}{z}$.

b) Solve $\frac{dx}{x(y^2+z)} = \frac{dy}{-y(x^2+z)} = \frac{dz}{z(x^2-y^2)}$.

OR

9. a) Express $\vec{f} = 2xi - 2y^2j + xzk$ in cylindrical coordinates system and find f_ρ, f_ϕ, f_z.

b) Express $\vec{f} = xi + yj + zk$ in spherical coordinate system and find f_r, f_θ, f_ϕ.

PART – D

Answer **two full** questions. **(2×10=20)**

10. a) Form the partial differential equation by eliminating arbitrary functions from $lx + my + nz = \phi\,(x^2+y^2+z^2)$.

b) Solve $x(1 + y)p = y\,(1 + x)\,q$.

OR

11. a) Solve $(D^2 - 5DD' + 4D'^2)\,z = \sin(4x + y)$.

b) Solve $p^2 = z^2\,(1 - pq)$.

12. a) Solve by Charpits method $z^2(p^2 + q^2 + 1) = 1$.

b) Solve $[D^2 - DD' - 6(D')^2]z = xy$.

OR

13. a) $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ given $u(0, t) = 0$, $u(l, t) = 0$, $u(x, 0) = k(lx - x^2)$, $\left(\frac{\partial u}{\partial t}\right)_{t=0} = 0$.

b) Solve $\frac{\partial u}{\partial t} = 16\frac{\partial^2 u}{\partial x^2}$, given $u(0, t) = 0$, $u(1, t) = 0$, $\forall t$, $u(x, 0) = x^2 - x$, $0 \le x \le 1$.